EXTENSION OF SIMPLE ONE-ZONE MODELS FOR RADIATIVE HEAT TRANSFER IN OVENS AND FURNACES

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Results are established and interpolated for heat transfer for models of limiting type with regard to surface configuration and bulk emission spectrum.

1. Introduction. Hottel [1] has indicated that there are models extreme in surface configuration and has proposed interpolation formulas for heat transfer for intermediate models with a gray spectrum for the medium. Here analogous results are presented for an antigray spectrum of the medium. It has been pointed out [2-4] that any real spectrum falls in the range between gray and antigray as regards the heat-transfer rate. Therefore, the results may be extended by recommending working formulas of interpolation type as regards body configuration and surface radiation spectrum.

2. Six Characteristics of Simple One-Zone Models. 2.1. A system of bodies is represented as three isothermal ones: F_0 and F_* with gray surfaces and the bulk of the real medium without allowance for scattering. As the bulk is not divided into zones, while its properties are decisive, such models are called one-zone ones.

2.2 The surfaces may be divided into elements or parts and have different shapes such as those shown in Fig. 1. The spatial figures may be approximated as spheres, cylinders, cubes, parallelepipeds, etc. Figure 1 is readily supplemented with a set of intermediate models ranging from the extreme one 1 with a uniform distribution of the elements dF_0 and dF_* over a sphere to continuous concentric spheres or coaxial long cylinders, which may be called models 2 and 3 in accordance with the type of internal surface. Hottel considered figures 2' and 3' of [1] as extreme models, which are similar to a metallurgical or heating furnace, whereas figure 1 is an idealized model for a boiler furnace. Form 3', which might appear exotic, was used in [5] to simulate a rotating furnace. The concave lining becomes a heating surface as soon as it is freed from the mixture as the furnace rotates. The planar surface of the filling on the other hand resembles the lining at the same time.

2.3. The surface F_* is considered as an adiabatic lining. More precisely, it is assumed that the convective heat flux is equal to the thermal loss for it.

2.4. One of the simplifications in the single-zone model is that one uses angular coefficients calculated in the absence of the medium (the transmission of this is extracted from the integrals). The formulas for the coefficients are also simplified. For example, the self-irradiation coefficient for the concave surface F_* with surface F_0 planar is calculated as $\varphi=1-F_0/F_*$. Strictly speaking, this is permissible only for model 2 when the bodies are completely concentric or coaxial, or else for a plane-parallel layer as a particular case of model 2 when the radii of the bodies are infinite. In what follows we use the characteristic $C = F_0/(F_0 + F_*)$.

2.5. The surface radiation of the bulk is described by the gray or antigray spectra shown in Fig. 2. The antigray spectrum consists of bands and lines with square shapes, where the spectral absorptivity is one within them and zero between them. The concept of antigray means that it is opposite not only in form in Fig. 2 but also in heat-transfer characteristics. All real spectra lie in the range between gray and antigray as regards form and the heat fluxes at the surfaces or the temperature of the adiabatic lining obtained incidentally in solving the problem. The advantages of using gray and antigray spectra, which strictly speaking do not occur in practice, are increased in relation to the maximal simplification of the calculations, particularly for the antigray spectrum. In particular, both

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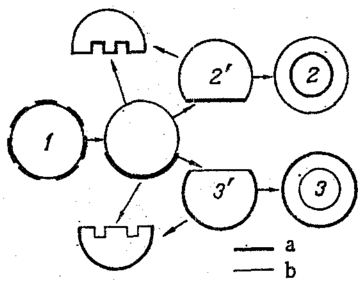

Fig. 1

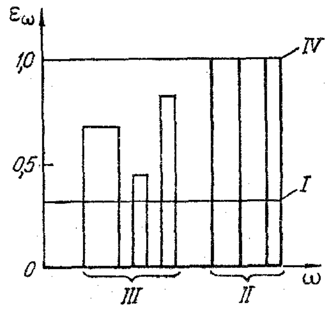

Fig. 2

Fig. 1. Transformation of one-zone models as regards body configuration: a) heating surface F_0; b) lining surface F_*.

Fig. 2. Simple models for the surface-radiation spectrum for the bulk: I) gray; II) antigray; III) rectangular; IV) black.

spectra enable one to use zonal equations, whereas the solution for any real spectrum is obtained as infinite series, which have so far been derived only for model 1 [3] and for a plane-parallel layer [6]. One is justified in using the extreme spectra not only by the simplifications. It is found that in most cases similar results are obtained and interpolation does not produce a substantial error.

2.6. The differences in degree of blackness and absorptivity of the bulk are incorporated only in the first-order passage of the rays through the bulk. Then the density of the resultant flux at F_0 is written as

$$q_{p0} = A_0 K \sigma (\varepsilon_1 T^4 - a_1 T_0^4). \tag{1}$$

The multiple-reflection coefficient is extracted from the parentheses. One uses the degrees of blackness ε_i or the absorptivities a_i in the formulas in accordance with the disposition of the sources. All the formulas below are written in terms of degrees of blackness for the case of fuel burning in the bulk, when $T > T_0$.

3. Formulation. The essence of the treatment is to incorporate multiple reflections together with the real bulk spectrum. One specifies the surfaces F_0 and F_*, the configurations of these, the optical characteristics of the bodies A_0, A_*, ε_i, a_1, and the temperatures T of the medium and T_0 of the heating surface. The q_{p0}, the density of the resultant flux at the heating surface, is given by (1). The temperature T_* of the lining surface is found incidentally from the condition $q_{p*} = 0$. The task amounts to determining the coefficient K in (1).

4. Multiple Reflection Coefficient. The quantities K, K', and K'' in (1) denote the coefficients for the actual spectrum of the surface radiation from the bulk and for gray and antigray ones. The subscript 1-2 denotes the interpolation formula for the extreme models 1 and 2 involving the use of the interpolation factor $M_{12}(S)$. The corresponding meaning attaches to K_{1-3} and $M_{13}(S)$. The interpolation number $1 \geqslant S \geqslant 0$ takes the extreme values $S = 1$ for model 1 and $S = 0$ for models 2 or 3 and 2' or 3'.

5. Body-Configuration Interpolation with a Gray Bulk. This case has been considered in detail in [1]. The chain of models 1 → 2 is described by

$$K'_{1-2} = \frac{1}{\varepsilon_1 + M'_{12} A_0 C (1 - \varepsilon_1)}, \quad M'_{12} = \frac{1 - \varepsilon_1 - S(C - \varepsilon_1)}{1 - C\varepsilon_1 - SC(1 - \varepsilon_1)}. \tag{2}$$

For the ideal model 1 for a boiler furnace, $S = 1$, we get $M' = 1$:

$$K'_1 = \frac{1}{\varepsilon_1 + A_0 C (1 - \varepsilon_1)}. \tag{3}$$

Hottel notes that formula (3) was known in 1928 or even earlier.

For $S = 0$

$$M'_2 = \frac{1 - \varepsilon_1}{1 - C\varepsilon_1}, \quad K'_2 = \frac{1 - C\varepsilon_1}{\varepsilon_1 (1 - C\varepsilon_1) + A_0 C (1 - \varepsilon_1)^2}.$$

TABLE 1. Grayness Parameters for a Layer of the Medium with Optical Thickness x_0 Bounded by a Black Cold Surface and an Adiabatic Gray Surface with Specular Reflection for Any Reflection Coefficient R_*, t = 1000°C

x_0, cm·atm	Carbon dioxide	Water vapor	x_0, cm·atm	Carbon dioxide	Water vapor
0,1	0,62	0,92	20	0,25	0,42
0,5	0,36	0,79	30	0,25	0,37
1	0,32	0,72	100	0,25	0,33
5	0,31	0,54	300	—	0,29
10	0,25	0,49			

This result was derived by Nevskii and Timofeev in 1934 [7]. The properties of a gray spectrum give us that $(1-\varepsilon_1)^2 = 1-\varepsilon_2$ and for an oven model $C=(1-\varphi)/(2-\varphi)$, which gives

$$K_2' = 1 + \frac{1-\varepsilon_1 + R_0(1-\varphi)(1-\varepsilon_2)}{1-\varphi(1-\varepsilon_1) - R_0(1-\varphi)(1-\varepsilon_2)}. \tag{4}$$

This result is derived only by the method of multiple reflections [2]. Formula (4), which uses ε_2, is more accurate. At the same time, the form $K = 1 + \delta$ includes the addition δ, which is always positive, to the heat flux due to the reflections from the surfaces.

The model chain 1 → 3 is described by (2) also, but with the interpolation factor

$$M_{13}' = \frac{\varepsilon_1 + (1-\varepsilon_1/C)\,[1-S(1-C)]}{C\varepsilon_1 + (1-\varepsilon_1)\,[1-S(1-C)]}. \tag{5}$$

With S = 1 we get $M_1' = 1$ and correspondingly formula (3) for a boiler furnace. For S = 0 we get a further extreme result due to Hottel [1]:

$$M_3' = \frac{1-\varepsilon_1(1-C)/C}{1-\varepsilon_1(1-C)}. \tag{6}$$

6. Body Configuration Interpolation with an Antigray Bulk. The model chain 1 → 2 is described by

$$K_{1-2}'' = \frac{A}{A - M_{12}''A_*(1-C)(1-\varepsilon_1)}, \quad M_{12}'' = \frac{A}{A-(1-S)\,A_0A_*C\varepsilon_1}. \tag{7}$$

For an ideal boiler-furnace model we have S = 1 and $M_1'' = 1$:

$$K_1'' = 1 + A_*\,\frac{(1-C)(1-\varepsilon_1)}{A_0C + A_*\varepsilon_1(1-C)}. \tag{8}$$

The result was published in [3]. For an oven model S = 0

$$M_2'' = \frac{A}{A - A_0A_*C\varepsilon_1}, \quad K_2'' = 1 + A_*\,\frac{(1-C)(1-\varepsilon_1)}{A_0C + A_*\varepsilon_1(1-C-CA_0)}. \tag{9}$$

This result was published in [2].

The model chain 1 → 3 is described by

$$K_{1-3}'' = 1 + A_*\,\frac{(1-C)(1-\varepsilon_1)}{A_0C + M_{13}''A_*\varepsilon_1(1-C)}, \quad M_{13}'' = R_0 + A_0S. \tag{10}$$

For an ideal boiler-furnace model S = 1, we get $M_1'' = 1$ and correspondingly (8). For the oven model 3, S = 0, $M_3'' = R_0$:

$$K_3'' = 1 + A_*\,\frac{(1-C)(1-\varepsilon_1)}{A_0C + A_*R_0\varepsilon_1(1-C)}. \tag{11}$$

This particular result is derivable from the formulas of [2]. The coefficients S for the different models may be taken as independent of the spectrum and extracted from [1].

7. Conclusions from Sections 5 and 6. Formulas (3), (4), (6), (8), (9), (11) for the quantities K_1', K_2', K_3', K_1'', K_2'', K_3'' can be derived from the zonal equations or by the multiple-reflection method. They define the extreme results for the model chains 1 → 2 and 1 → 3, on which the interpolation methods are based. One can interpolate the results in terms of body configurations from (2), (5), (7), (10), which define K_{1-2}', K_{1-3}', K_{1-2}'', K_{1-3}''. These quanti-

ties, whose subscripts are omitted below, are used in interpolating the results with respect to the spectrum.

8. Result Interpolation on Bulk Spectrum. A simple linear interpolation was used in [2-4]:

$$K = K'' + (K' - K'')\,c, \tag{12}$$

where $K' \geqslant K''$ always; it was recommended that the interpolation factor should be calculated from

$$c = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 (1 - \varepsilon_1)}, \tag{13}$$

according to which it has the meaning of the ratio of the emissivity of the bulk for the inherent and blackbody radiations from it. Formula (13) is exact for a black cold surface F_0 for any value of A_*. Values of c are given in Table 1 for the most important components of combustion products, which agree with the data of [8] for specular reflection from the lining.

The entire set of real spectra can be arranged in a series intermediate in relation to the gray and antigray ones. Factor c increases as one approaches the gray spectrum, and therefore it may be called the medium grayness parameter. A difficulty is that c is dependent not only on the composition of the medium but also on the dimensions and shape of the bulk.

9. Antigray Spectrum Extrapolation. In [9] we find a method of calculation that has become fairly widely used, particularly in the German literature. We consider it a particular case based only on the antigray spectrum. Multiple reflections within the bands of the real spectrum are incorporated indirectly by increasing the effective thickness of the bulk in accordance with $l = l_e/A^{0.85}$.

In our treatment, one should use all the formulas of section 6, in which ε_1 is calculated from the effective thickness l. It has been shown [9] that the method as a rule gives an error of less than 5%, but in one case an error of 16% was found.

10. Gray-Spectrum Extrapolation. Hottel maximally simplified his spectrum model to a gray gas alone and a transport gas alone. In that case, one uses the weight of the gray gas, which is dependent on the optical thickness in inexplicit form from

$$\varepsilon_0 = \varepsilon_1^2/(2\varepsilon_1 - \varepsilon_2). \tag{14}$$

In the proportion ε_0 of the black spectrum, the medium is considered as gray, while in the residual proportion it is considered as transparent. The value of ε_0 was especially considered in [10]. If one derives general formulas for K, then at the limits $\varepsilon_0 \to 1$ or $\varepsilon_0 \to \varepsilon_1$ one gets the quantities K' and K'' for the gray and antigray spectra. Unfortunately, the general formula was written only for a simple oven model 1 [11]:

$$\frac{1}{K_1} = CA_0 \left[1 - \frac{\varepsilon_1}{\varepsilon_0} - \frac{\varepsilon_1}{A}\left(1 - \frac{1}{\varepsilon_0}\right)\right] + \varepsilon_1.$$

The limiting values are obtained correctly. An approximate value may be used for ε_0 in this formula, which may be calculated in a way different from (14). Below we use another more accurate quantity D, which is derived by summing the infinite series that incorporate multiple reflection.

For oven model 2, Hottel gives a formula only for a lining that completely reflects the fluxes, $A_* = 0$:

$$\frac{1}{K_2} = \frac{\varepsilon_1}{\varepsilon_0} R_0 + A_0 \frac{1}{1 + \dfrac{1}{1 - \varphi + \varepsilon_1/(\varepsilon_0 - \varepsilon_1)}},$$

where $1 - \varphi = F_0/F_*$.

For $\varepsilon_0 \to 1$ we get the K_2' of section 5. This result is independent of A_*. In the antigray spectrum limit $\varepsilon_0 \to \varepsilon_1$, K_2'' is dependent on A_*, and therefore the formula on the whole is a particular one. Other and more general formulas for this spectrum model are excessively complicated. It is therefore undesirable to use this extrapolation method for very simple models.

11. Comparison of Spectrum-Interpolation Methods. We compare the methods of sections 8 and 9 on two examples. In the first, as in [8], we consider a planar layer bounded by a black cold heating surface and an adiabatic lining. From our method

$$K' = 2 - \varepsilon_1, \quad K'' = 1 + A_*(1 - \varepsilon_1).$$

Interpolation from (12) and (13) gives the exact result

$$K = A_*(2 - \varepsilon_1) + R_*\varepsilon_2/\varepsilon_1.$$

The comparison with method 9 should be made for a degree of blackness in the oven or furnace space $a_f = A_0\varepsilon_1 K$, since ε_1 in that case is calculated from the effective thickness $l = l_e/A^{0.85}$. In the case of a black lining, $A_* = 1$, and all the methods give the same result $a_t = A_0\varepsilon_1(2 - \varepsilon_1)$, where ε_1 is calculated from l_e. The largest discrepancies are to be expected when there is the maximum contribution from multiple reflection, i.e., when $A_* = 0$. Then $a_{t8} = A_0\varepsilon_2(l_e)$ and $a_{t9} = A_0\varepsilon_1(l)$, where $l = l_e \cdot 1.8025$. The discrepancies increase as the optical thickness of the bulk decreases, i.e., as the contribution from multiple reflections increases, and in the limit they attain 10%.

In the second case, we consider the ideal model for a boiler furnace 1, this being the unique case for which one can write the multiple-reflection series without considering the layer as planar. According to [3]:

$$K_1 = \frac{A}{A(1 - RD) - A_*(1 - C)(1 - RD - \varepsilon_1)}, \tag{15}$$

where the emissivity averaged over the fluxes for the bulk is written approximately for the inherent radiation:

$$D = \sqrt{(\varepsilon_3 - \varepsilon_2)/\varepsilon_1}.$$

Formula (15) enables one to check both interpolation methods. The quantities a_t' and a_t'' are obtained with $D = 1 - \varepsilon_1$ and $D = 0$ correspondingly. For simplicity we put $A_0 = A_*$ and $C = 0.5$. Then

$$a_t' = A_0 \frac{2\varepsilon_1}{A_0 + \varepsilon_1(2 - A_0)}, \quad a_t'' = A_0 \frac{2\varepsilon_1}{1 + \varepsilon_1}.$$

On our method

$$a_{t8} = A_0 \frac{2}{1 + \varepsilon_1}\left[\varepsilon_1 + \frac{R_0(\varepsilon_2 - \varepsilon_1)}{A_0 + \varepsilon_1(2 - A_0)}\right]. \tag{16}$$

On the method of section 9

$$a_{t9} = A_0 \frac{2\varepsilon_1(l)}{1 + \varepsilon_1(l)}, \text{ where } l = l_e/A^{0.85}. \tag{17}$$

The exact formula is obtained from (15):

$$a_t = A_0 \frac{2\varepsilon_1}{1 + \varepsilon_1 - R_0 D}. \tag{18}$$

The results of (16)-(18) coincide if the surfaces are black, as should be so in any case. The discrepancies are largest when the multiple reflections have the largest effect. For this we consider the limit $\varepsilon_1 \to 0$. Then $(\varepsilon_2 - \varepsilon_1) \to \varepsilon_1(1 - \varepsilon_1)$, $D \to (1 - \varepsilon_1)$, and our approximation of (16) coincides with the accurate result (18):

$$a_t = A_0 \frac{2\varepsilon_1}{A_0 + \varepsilon_1(2 - A_0)}.$$

The relative error of method 9 increases as A_0 decreases. In the limit $A_0 \to 0$, $a_t \to A_0$

$$a_{t9} \to A_0 \frac{2\varepsilon_1(l \to \infty)}{1 + \varepsilon_1(l \to \infty)}.$$

As $\varepsilon_1(l \to \infty) \to 1$, method 9 does not introduce a large absolute error. We give the numerical values: $A_0 = 0.1$, $x_e = 0.1$ cm·atm. For carbon dioxide at t = 1000°C we get $\varepsilon_1 = 0.01$, $\varepsilon_1(l) = \varepsilon_1(0.078) = 0.039$. The approximation of (17) gives $a_t = 0.075A_0$ instead of the accurate value $a_t = 0.168A_0$. The relative error of method 9 is 55%.

NOTATION

a_1, a_2, a_3, volume absorptivities for the black incident flux for single, double, and triple passage through the volume; ε_1, ε_2, ε_3, volume emissivities under the same conditions; a_f, emissivity of the furnace space; q_r, resultant flux density, W/m²; K, composite factor for multiple reflection from the envelope; A, absorption coefficient (emissivity); $R = 1 - A$; F, surface area, m²; c, interpolation coefficient; M and S, the same for interpolation over the configuration. Subscripts: 0, *, heating and lining surfaces, respectively; 1, 2, 3, extreme models for configuration. Primes: values of gray and antigray surface-radiation spectra of the bulk medium.

LITERATURE CITED

1. H. C. Hottel, "First estimates of industrial furnace performance: the one-gas-zone model reexamined," Heat Transfer in Flames, N. H. Afgan and J. B. Beer (eds.), Scripta Book Company, Washington DC (1974), pp. 5-28.
2. S. P. Detkov, "Heat transfer in gas ovens," Inzh.-Fiz. Zh., 43, No. 2, 244-249 (1982).
3. S. P. Detkov, "Heat exchange by radiation in a furnace," Teploenergetika, No. 5, 75-76 (1981).
4. S. P. Detkov, "Engineering formulas for heat transfer in gas ovens," Izv. Vyssh. Uchebn. Zaved., Chern. Metall., No. 8, 112-117 (1979).
5. S. P. Detkov and O. A. Bryukhovskikh, "The initial period of heating for a massive body in an oven at a high temperature," Inzh.-Fiz. Zh., 45, No. 2, 292-296 (1983).
6. S. P. Detkov, "Heat exchange by radiation between a nongray gas and gray surfaces," Inzh.-Fiz. Zh., 28, No. 1, 5-11 (1970).
7. A. S. Nevskii, Radiative Heat Exchange in Ovens and Furnaces [in Russian], Metallurgiya, Moscow (1971).
8. A. S. Nevskii, "The effects of lining reflectivity on radiative heat exchange," Teplofiz. Vys. Temp., 11, No. 5, 1031-1036 (1973).
9. G. Wolter, H. Schneider, and R. Steinhardt, "Naherungs-formelm zur Berechnung der Strahlungswarme-übertragung in brennstoffbeheizten Industrieofen," Energietechnik, 30, No. 12, 461-465 (1980).
10. S. P. Detkov, "A characteristic of gas radiation with a complete spectrum," Inzh.-Fiz. Zh., 18, No. 4, 638-642 (1970).
11. H. C. Hottel and A. F. Sarofim, Radiative Transfer, McGraw-Hill, New York (1967).

A METHOD OF MEASURING THE SHEAR AND ROTATIONAL VISCOSITY OF MAGNETIC FLUIDS

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The possibility of determining the shear and rotational viscosity of a magnetic fluid in a coaxial viscosimeter by means of two similar measurements of the rotational velocity of the inner cylinder in the dynamic regime is given a foundation.

1. The shear state of a magnetic fluid (MF) is determined by two viscosity coefficients, shear η, and rotational η_r. Up to now, the question of a simple method to measure the shear and rotational viscosity in one experiment remained urgent. A method is proposed in [1] for the determination of η and η_r in a coaxial viscosimeter by measuring the velocity of the inner cylinder and the friction moment of the outer cylinder in the steady-state regime. The advantage of the method is that the measurements are performed only in the stationary regime. However, measurement of the velocity and friction motion requires different realizations of the method from the accuracy and apparatus viewpoints, which is a substantial disadvantage.

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